TEKS G.3 D
Symmetry

(G.3) Coordinate and transformational geometry. The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). The student is expected to:

(A) describe and perform transformations of figures in a plane using coordinate notation;

(B) determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane;

(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane; and

(D) identify and distinguish between reflectional and rotational symmetry in a plane figure.

Overview

This lesson provides students with insight in how symmetry is used in real life, such as in classifying biological organisms and designs. Transformation concepts that are explored include reflectional and rotational symmetry, and the order of symmetry.

Instructional Strategies

Students are engaged in guided exploration and discovery through cooperative learning, whole class, and independent activities.

Lesson Objectives

- Students understand rotational and reflectional symmetry.
- Students understand how symmetry can be used as an organizational principle in biology and in mathematics.

For **Teacher's Eyes Only**

Research shows that most students entering the high school geometry course are operating at or below the van Hiele visual level of understanding (Hoffer, 1983; Shaughnessy & Burger, 1985). Although these students are able to identify different shapes, they may find it hard to recognize and reason with specific characteristics of shapes. Students at this level are often not yet ready for the abstract reasoning required in geometry courses.

Transformations can provide a bridge between the initial visual intuition of the students and the more formal reasoning of the higher van Hiele levels. All students stand to benefit from an integration of transformation geometry in the high school geometry course. (Coxford & Usiskin, 1971, 1975 and Okolica & Macrina, 1992).

This lesson provides an opportunity to coordinate topics in mathematics and biology. Most organisms have some kind of symmetry. Some have bilateral symmetry (a butterfly), while others have rotational symmetry (a jellyfish). Very few plants and animals have no symmetry at all (sponges). Likewise, mathematical figures (for example, certain quadrilaterals) can have reflectional and/or rotational symmetry.

REFERENCES

Coxford, A.F. & Usiskin, Z.P. (1971 & 1975). *Geometry – A Transformation Approach*. River Forest, IL: Laidlaw Brothers.

Hoffer, A. (1983). Van Hiele-based research. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (205-227). New York: Academic Press.

Okolica, S. & Macrina, G. (1992). Integrating transformational geometry into traditional high school geometry. *The Mathematics Teacher*, 85(9), 716 – 719

Shaughnessy, J.M., & Burger, W.F. (1985). Spadework prior to deduction in geometry. *The Mathematics Teacher,* 78(6), 419-428.

Misconceptions

☒ Misconception

There is some evidence that students may confuse reflectional and rotational symmetry (Son, 2006). In particular, a student may confuse reflectional symmetry and 180 degrees rotational symmetry.

REFERENCE
Son, J. (2006). Investigating preservice teachers' understanding and strategies on a student's errors of reflective symmetry. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková, *Proceedings of the 30th annual conference of the International Group for the Psychology of Mathematics Education,* pp. 145-152

☑ Mathematics Concept

A figure has **reflectional symmetry** if it can be reflected across a line so that the image coincides with the pre-image. The line of symmetry divides the figure into two congruent halves.

A figure has **rotational symmetry** if it can be rotated about a point so that the image coincides with the pre-image.

🛠Rebuild Concept

The pictures of the plants and animals can be used to explain the difference between rotational and reflectional symmetry. For example, the butterfly and the lobster both have reflectional symmetry. However, they do not have 180 degrees rotational symmetry. If you turn the card 180 degrees around, the head is pointing in the opposite direction.

Student Prior Knowledge

Students need some prior knowledge of reflections and rotations. Students' prior experiences with transformational geometry in grade 8 should be sufficient (TEKS grade 8 objective 8.6).

Materials

Each student needs the following set of materials:

- a set of six cards with pictures of plants and animals
- equilateral triangle templates
- square templates
- a pair of scissors
- pencil or pen
- Poster board or butcher paper

The teacher needs the following additional materials:

- 6 to 8 large triangle templates
- 10 to 12 large square templates
- black marker
- adhesive tape

5 E's

ENGAGE

The learner is introduced to a new experience and must draw from prior experiences to make sense of the engage activity.

In this engage activity each pair of students works with a set of cards (see the black line masters for the student materials) showing a picture of a plant or an animal. Using their prior knowledge, they group the organisms according to a personal criterion. At first, the students may classify the plants and animals based on biological principles. For example, some of the organisms live on land and others live in the sea. You should allow the students room for personal exploration in this phase.

Once a student has completed their classification, challenge the students to group the plants and animals according to their shape. You could ask a students,

- Can you classify the plants and animals according to how they are shaped?
- Can you classify the organisms according to their symmetries?

Based on their body plan, the plants and animals can be placed in three categories: bilateral symmetry (butterfly, lobster), radial symmetry (apple, jelly fish, sea anemone), and asymmetric (sponge). In the explore and elaborate phases, students will find create designs of equilateral triangles and squares which also possible different kind of symmetries.

EXPLORE

During the explore activity, the student becomes directly involved with a particular phenomena by manipulation of materials that are used to discover the phenomena.

In the explore phase, the students create symmetric designs based on an equilateral triangle. Group the students in pairs. Each pair needs several blank equilateral triangle templates. Students need to cut out the templates to check their designs and to organize them later on.

The equilateral triangle on the template is divided into 6 parts. Coloring one or more parts black creates a pattern. Although there are $2^6 = 64$ possible patterns, many are in fact the same. Not all patterns are symmetric. All in all there are 6 different symmetric designs.

In the explore phase, students can discover different symmetric designs on their own. If you notice that a pair has produced a symmetric design, hand them a copy of the large triangle template. The pair can reproduce their design, and post it in front of the classroom, where everyone can see their designs. Solicit a contribution from as many different pairs as possible.

Start the explain phase as soon as all 6 designs are posted in front of the classroom and the students feel that there are no more symmetric designs to discover.

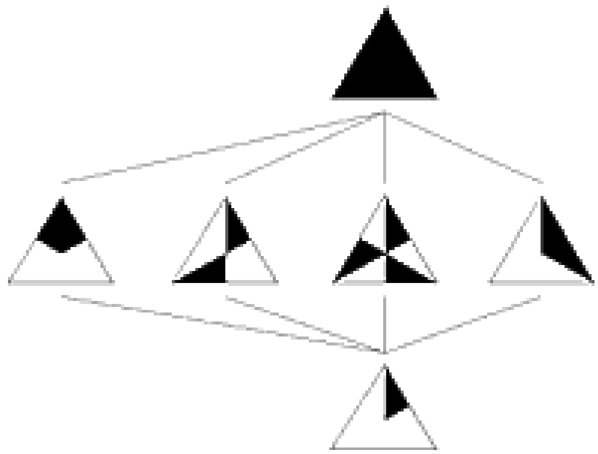
EXPLAIN

The student communicates in verbal and written form about the information derived from the learning experience.

In the explain phase, the class as community describes the symmetries of each design formally, and classifies the different designs based on their symmetries.

Ask several students to describe the symmetries of a particular design. You may have them write down the types symmetries next to each large template. Encourage students to be as specific as possible. For example, have a student indicate all lines of symmetry or the measure of rotational symmetry. Guide students in using the correct terminology.

Group the designs according to their symmetries. Compare the symmetries of different designs. Some designs possess all the symmetries of another design, plus some more. In a sense, the former design is more symmetrical than the later. Indicate this relationship in a hierarchical picture. A possible way to organize these designs is shown in the following figure (Park, 2001):



The top-most design is completely symmetric. The other designs have rotational symmetry.

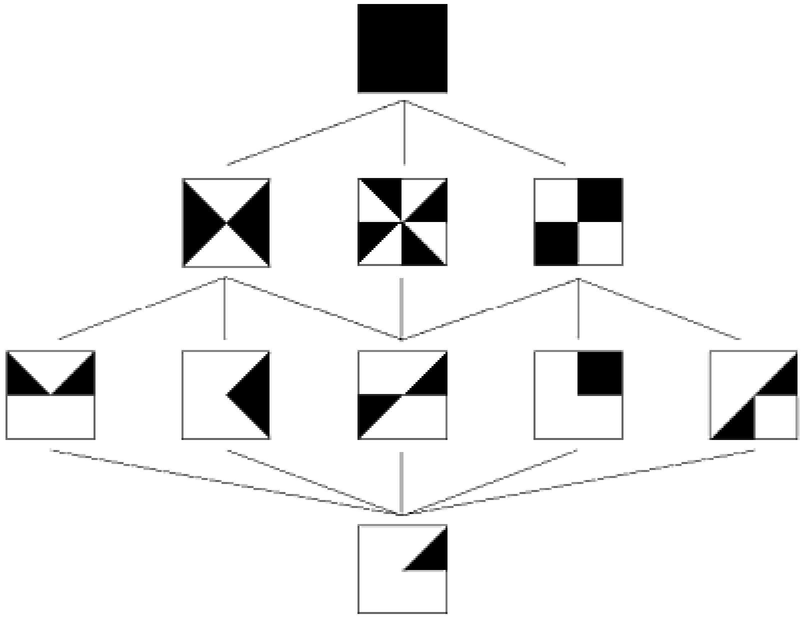
REFERENCE
Park, J. (2001). Analysis and Synthesis in Architectural Designs: A Study in Symmetry. *Nexus Network Journal,* 3(1). Retrieved February 25, 2009 from http://www.emis.de/journals/NNJ/Park.html

ELABORATE

During the elaboration phase, students expand their knowledge by making connections about what they have learned and applying this new knowledge to real world situations.

To elaborate their knowledge of reflectional and rotational symmetry, the students repeat the explore activity but with a square instead of an equilateral triangle. The symmetries of a square are more complicated than that of an equilateral triangle.

Each pair of students creates a poster showing their designs. There are 10 possible symmetric designs. A possible way to organize the different designs is the following:



🌡 EVALUATE

Evaluation throughout the learning experience is an ongoing process and has a diagnostic function.

At the end of the lesson the students revisit their classification of plants and animals. The students should now be able to accurately describe the symmetries of each organism and place the organisms into logical groups based on their body plan. Finally, the students create a concept map to organize their new knowledge of symmetries in a meaningful way.

Black-Line Masters

Symmetry of Triangles

1. Like plants and animals, geometric designs and figures can have different kinds of symmetry. Consider the blank templates of equilateral triangles. Color some parts of the equilateral triangles black to create a symmetric design. What kind of symmetry does your design have? Try to create as many different designs as possible. Describe the symmetry of each of your design.

2. Cut out each of your designs. Place your designs into logical groups. What principle did you use?

Symmetry of Squares

1. Consider the blank templates of squares. Color some parts of the equilateral triangles black to create a symmetric design. What kind of symmetry does your design have? Try to create as many different designs as possible. Describe the symmetry of each of your designs.

2. Cut out each of your designs. Place your designs into logical groups. What principle did you use? Arrange your designs on a poster aimed at the other students in the class.

3. Consider the plants and animals pictured on the cards again. Describe all of the symmetries of each organism as accurately as possible. What is the order of each symmetry? Classify the organisms according to the symmetry of their body plan.

Reflecting on Symmetry

4. Create a concept map of the mathematical concept of symmetry. Include as examples some of the plants, animals, and the symmetric designs you have created.

Consider the plants and animals on the cards. They are alike in some aspects, and different in others. Place the organisms into logical groups. What principle did you use? Group them according to a different principle.